Trial Risk Assessment Based on Total Utility of Generalized Fuzzy Numbers

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***Abstract*—In this paper, we propose a risk assessment method based on total utility of generalized fuzzy numbers (GFNs) towards trial process management. Firstly, the possible risk events in trial business process are sorted out according to trial experience and expertise of law experts. Then the occurrence probability, impact degree and repairability are used as evaluation factors and we transform corresponding evaluation descriptions into form of GFNs. Then multiple experts' evaluations will be aggregated to get fuzzy risk value. Finally, we de-fuzzy the fuzzy risk value by calculating total utility of GFNs, achieving the purpose of risk assessment. Proposed method mainly solves two core problems. The first is that we use GFNs to express and calculate fuzzy risks after analyzing risk events, avoiding subjectivity and uncertainty in traditional risk assessment methods. The second is that we de-fuzzy the fuzzy risk value by calculating total utility of GFNs based on Maximum and Minimum Set, obtaining exact risk value as the ranking basis for accurate risk assessment. We have applied proposed method into real trial cases to realize the evaluation of trial risks for judicial business, and the results demonstrate accuracy, effectiveness and applicability of proposed method in trial scenes.**

***Index Terms*—risk assessment, generalized fuzzy number, total utility, trial process management**

I. Introduction

With the establishment of modern judicial concept and the rapid development of "smart court" [1], trial risk has gradually attracted more attention of judicial administrators and all walks of life. As a supervision mechanism oriented to whole trial business, trial risk management should cover more comprehensive evaluation objects and trial procedures so as to build rational risk assessment and warning system based on accurate risk assessment results. And the key lies in how to identify multi-factor trial risks and establish a quantitative risk assessment model [2].

Among existing risk assessment methods, risk prediction approaches based on machine learning mechanism mainly depend on a large amount of historical data [3], [4]. For example, Yala A et al. [5] propose to extract data representation based on trial documents to establish the risk prediction model. However, most of the trial systems just provide outcome documents such as trial transcripts and judgment documents and lack structured process data containing risk elements, which make it difficult to conduct semantic analysis and data annotation. Risk matrix evaluation methods [6] usually adopt multidimensional description language to localize the level of risk. Although some researches [7] bring in detectability and repairability to describe the detailed risk evaluations, it is still not completely out of the category of qualitative analysis and risk decomposed granularity is coarser, easily leading to large error of evaluation results. Poisson Process methods apply poisson distribution to describe the probability of certain events in unit time, probability prediction model based on which is usually strongly related with time series [8]. Therefore, it's more suitable for planning of road traffic capacity or investment strategies [9]. But for trial scenes whose process shall strictly follow judicial procedures and not as reference to time, it's lack of application value.

In view of the above, we propose a risk assessment method based on total utility of generalized fuzzy numbers (GFNs) towards trial process management. We first sort out possible risk events in the trial process. Then the occurrence probability, impact degree and repairability of each risk event are used as evaluation factors and described in form of GFNs. Then multiple judges or law experts' evaluations are aggregated to generate fuzzy risk value. Finally, the accurate risk value is obtained by calculating total utility of fuzzy risk value, achieving the purpose of accurate trial risk assessment.

In summary, we make the following contributions:

- GFNs are adopted to express evaluation factors instead of traditional, subjective and uncertain risk analysis descriptions. And we further establish self-adapting reasoning function to aggregate the evaluations of multiple judges.
- We calculate total utility of GFNs based on Maximum and Minimum Set to de-fuzzy the fuzzy risk value so as to obtain an accurate risk value which is able to completely distinguish different level risks.
- We evaluate the performance of proposed method with real trial cases. And the results demonstrate the accuracy, effectiveness and business applicability of proposed method in trial risk assessment.

The remainder of the paper is structured as follows. We present some preliminaries in Section II. Then, we elaborate the design of proposed method in Section III. We illustrate case study results in Section IV and conclude in Section V.

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II. Preliminaries

In this section, we bring up some basic concepts of fuzzy numbers, including the definition and extended form of fuzzy numbers in Section II-A, the operation rules in Section II-B and the mapping strategy in Section II-C.

A. Definition

According to fuzzy mathematics theory, general fuzzy number is a generalization of real number. Suppose that u is an universe of discourse, A is a fuzzy set on U when the following condition is satisfied:

$$A = \{(x, \mu_A(x)) : \forall x \in U, \exists \mu_A(x) \in [0,1]\}, \tag{1}$$

where $\mu_A(x)$ represents membership function of element x in U to fuzzy set A. When the fuzzy set is bounded, A is a general fuzzy number, usually expressed in interval number form as $A = (a, b, c, d; w)$, where w represents the weight of generalized fuzzy number. And the membership function $\mu_A(x)$ is defined as follows:

$$\mu_A(x) = \begin{cases} \frac{w(x-a)}{b-a}, & if \quad a \le x \le b \\ w, & if \quad b \le x \le c \\ \frac{w(d-x)}{d-c}, & if \quad c \le x \le d \\ 0, & otherwise \end{cases} \tag{2}$$

GFN can be illustrate as a graph on axes. With different interval values and weights, GFN can be extended into different forms. As shown in Fig 1 (I), when $w = 1$, A is a normal fuzzy number, and A is a trapezoidal fuzzy number when $a \le b < c \le d$. Fig 1 (II) presents that when $a = b$ and $c = d$, A is a rectangular fuzzy number, and when $a < b = c < d$, A is a triangular fuzzy number. Fig 1 (III) shows that A is a real number, a point on the x-axis, when $a = b = c = d$. When $a \ne d$ and $w = 0$, A becomes a interval, a line segment on the x-axis. And A is a line segment perpendicular to the x-axis, when $a = b = c = d$ and $w > 0$.

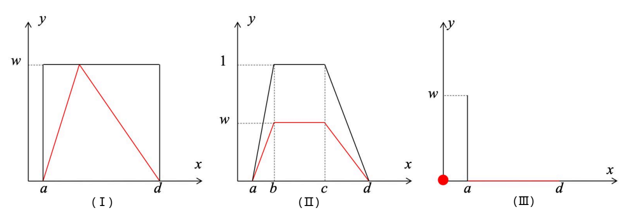

Fig. 1: GFNs in different forms.

B. Operation Rules

Most operations between fuzzy numbers follow uniform convention. In order to further standardize calculation results of fuzzy numbers, some researchers restrict the division algorithm specially, but core idea is still based on classical real number operation. In this paper, we adopts the operation rules proposed by Chen et al. [10]. Suppose there are two generalized fuzzy numbers: $A = (a_1, a_2, a_3, a_4; w_A)$ and $B = (b_1, b_2, b_3, b_4; w_B)$ where $a_1 \le a_2 \le a_3 \le a_4, b_1 \le b_2 \le b_3 \le b_4$ and they are both real numbers. $w_A, w_B \in [0, 1]$, then we define arithmetic operation rules as follows:

- Addition of GFNs:
$$A \oplus B = (a_1, a_2, a_3, a_4; w_A) \oplus (b_1, b_2, b_3, b_4; w_B) = (a_1 + b_1, a_2 + b_2, a_3 + b_3, a_4 + b_4; min(w_A, w_B))$$
- Subtraction of GFNs:
$$A \ominus B = (a_1, a_2, a_3, a_4; w_A) \ominus (b_1, b_2, b_3, b_4; w_B) = (a_1 - b_1, a_2 - b_2, a_3 - b_3, a_4 - b_4; min(w_A, w_B))$$
- Multiplication of GFNs:
$$A \otimes B = (a_1, a_2, a_3, a_4; w_A) \otimes (b_1, b_2, b_3, b_4; w_B) = (a_1 b_1, a_2 b_2, a_3 b_3, a_4 b_4; min(w_A, w_B))$$
- Division of GFNs:
$$A \oslash B = (a_1, a_2, a_3, a_4; w_A) \oslash (b_1, b_2, b_3, b_4; w_B) = (a_1/b_1, a_2/b_2, a_3/b_3, a_4/b_4; min(w_A, w_B))$$

C. Mapping Strategy

Facing the judicial scenario lacking of historical data, the primary problem of trial risk assessment is how to transform qualitative assessment into quantitative mathematical form and provide effective analytical data for subsequent risk aggregation and risk level determination. For this purpose, many researches adopt the 9-level mapping strategy proposed by Smucker et al [11]. As shown in Table I, the strategy defines 9 evaluation linguistic terms and requires decision makers to select evaluation words within specified range and convert them into corresponding generalized fuzzy numbers according to the mapping strategy. It is the most widely used conversion system in both research and application.

TABLE I: Mapping strategy between evaluation linguistic terms and generalized fuzzy numbers

Evaluation Linguistic Terms	Generalized Fuzzy Numbers
Absolutely Low (AL)	(0.0, 0.0, 0.0, 0.0; 1.0)
Very Low (VL)	(0.0, 0.0, 0.02, 0.07; 1.0)
Low (L)	(0.04, 0.1, 0.18, 0.23; 1.0)
Fairly Low (FL)	(0.17, 0.22, 0.36, 0.42; 1.0)
Medium (M)	(0.32, 0.41, 0.58, 0.65; 1.0)
Fairly High (FH)	(0.58, 0.63, 0.80, 0.86; 1.0)
High (H)	(0.72, 0.78, 0.92, 0.97; 1.0)
Very High (VH)	(0.93, 0.98, 1.0, 1.0; 1.0)
Absolutely High (AH)	(1.0, 1.0, 1.0, 1.0; 1.0)

III. Proposed Methodology

In this section, we will elaborate proposed methodology. The overall workflow will be illustrate in Section III-A, then we discuss the process of fuzzy risk aggregation in Section III-B and present the process of de-fuzzing fuzzy risk values by calculation of total utility in Section III-C

A. Overall Workflow

The overall workflow of proposed method is presented in Fig. 2. Combining with the trial process of historical cases, we first sorts out all kinds of trial risks and forms a general trial risk event database. It makes a relatively macroscopic risk analysis for trial process, avoiding excessive attention to trivial process nodes which may cause analysis results not

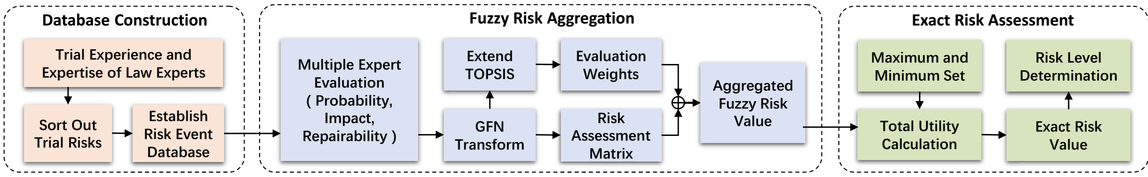

Fig. 2: The overall workflow of proposed method.

referable. In trial business, it usually contains a large number of judication-related provisions. At present, there is no targeted and effective text mining tool for trial process management, so only the general technology can be adopted for model training. However, the time cost of text preprocessing and model training is usually higher than that of manual combing, and the professionalism and accuracy must be lower than that of manual combing. On the other hand, different dimensions of risks need to be described after the risk events are sorted out, and it requires trial experience and expertise of law experts, which cannot be replaced just by computer technology.

As there are many factors may cause trial risk and lead to retrial, we define three core evaluation factors (risk probability, the degree of impact and repairability) to analyze trial risk. And we use definite mathematical language (generalized fuzzy numbers) to express these evaluation factors instead of traditional, subjective and uncertain risk analysis description. Then we establish self-adoptive fuzzy reasoning function, aggregating all these evaluations given by multiple law experts and calculating fuzzy risk value so as to ensure that final risk assessment is objective, comprehensive and accurate as far as possible. The details will be discussed in Section III-B.

Finally, we calculate total utility of the fuzzy risk value based on Maximum and Minimum Set so as to achieve the effect of de-fuzzing and obtain an accurate risk value which is able to completely distinguish different level risks, thus providing a more efficient and reliable basis for final risk ranking and assessment. The details will be elaborated in Section III-C.

B. Fuzzy Risk Aggregation

As mentioned in Section III-A, we first sort out the trial process of historical cases and establish a general trial risk event database based on trial experience and expertise of law experts. Then for a certain trial case, several law experts will be asked to evaluate each risk event in trial process respectively from three dimensions of probability, impact and repairability. And the optional description phrases are as shown in Table I. Multiple experts describe a risk event from three different dimensions to ensure comprehensive risk analysis and avoid human subjectivity as much as possible.

For each expert's evaluation, we transform risk description phrases into general fuzzy numbers according to the mapping strategy in Table I and construct fuzzy reasoning function to aggregate general fuzzy numbers which are transformed from three terms above. We define fuzzy reasoning function as follows:

$$RF = \frac{probability \otimes impact}{repairability}, \tag{3}$$

where $probability$, $impact$ and $repairability$ indicate probability, impact degree and repairability of risk event which are in form of general fuzzy numbers. And the operation rules between general fuzzy numbers have been discussed in Section II-B.

In order to further improve the objectivity of evaluation, we construct a weighted determination model based on extended TOPSIS (Technique for Order Preference by Similarity to an Ideal Solution) to synthesize evaluation results from multiple experts and calculate a more reasonable and objective fuzzy risk. More specifically, we first construct an evaluation matrix:

$$D = \begin{bmatrix} x_{11} & \cdots & x_{1n} \\ \vdots & \vdots & \vdots \\ x_{m1} & \cdots & x_{mn} \end{bmatrix}, \tag{4}$$

where n denotes the number of evaluation terms (n = 3 in this paper), m denotes the number of experts, and x_{ij} denotes generla fuzzy number corresponding to the evaluation given by the i-th expert on j-th evaluation term. Positive and Negative Ideal Solutions are as follows:

$$PIS = [v_1^+, v_2^+, \cdots, v_n^+], \tag{5}$$

$$NIS = [v_1^-, v_2^-, \cdots, v_n^-], \tag{6}$$

where we define v_i^+, v_i^- as:

$$\begin{cases} v_i^+ = min(x_{ji}) \\ v_i^- = max(x_{ji}) \end{cases} \quad 1 \leq j \leq m \tag{7}$$

Then we calculate Euclidean distance between each expert's evaluation vector $\mathbf{V}_i$ and the positive/negative ideal solutions respectively:

$$d_i^+ = d(\mathbf{V}_i, PIS) = \sqrt{\sum_{j=1}^{n} (x_{ij} - v_j^+)} \quad , \tag{8}$$

$$d_i^- = d(\mathbf{V}_i, NIS) = \sqrt{\sum_{j=1}^{n} (x_{ij} - v_j^-)} \quad , \tag{9}$$

Finally, we obtain the weight for each expert's evaluation vector as follows:

$$w_i^{'} = \frac{2}{d_i^+ - d_i^-}, \tag{10}$$

$$w_i = \frac{w_i^{'}}{\sum_{j=1}^{n} w_j^{'}}, \tag{11}$$

With weight for each expert's evaluation, we aggregate all evaluation results obtained from Equation 3 to calculate fuzzy risk value for the risk event:

$$\begin{aligned} Fuzzy\_Risk &= \sum_{i=1}^{m} w_i * RF_i \\ &= \sum_{i=1}^{m} w_i * \frac{probability_i \otimes impact_i}{repairability_i} \end{aligned} \tag{12}$$

C. De-fuzzy Fuzzy Risk Value

In order to obtain exact risk value for risk assessment, we need to de-fuzzy the fuzzy risk value $Fuzzy\_Risk$ which is in form of general fuzzy number $(a, b, c, d; w)$. Considering total utility of general fuzzy number, we de-fuzzy the fuzzy risk value based on Maximum and Minimum Set method, using total utility value to express exact risk value in trial process.

According to Equation 2, we denote $\mu_{\tilde{A}(x)}$ as the membership function of $Fuzzy\_Risk$. Then we define maximum set as $\tilde{M}$ and minimum set as $\tilde{G}$, the membership function of which are as follows:

$$u_{\tilde{M}}(x) = \begin{cases} (\frac{x-x_{min}}{x_{max}-x_{min}})^k, & x_{min} \leq x \leq x_{max} \\ 0, & otherwise \end{cases} \tag{13}$$

$$u_{\tilde{G}}(x) = \begin{cases} (\frac{x_{max}-x}{x_{max}-x_{min}})^k, & x_{min} \leq x \leq x_{max} \\ 0, & otherwise \end{cases} \tag{14}$$

where k represents the attitude of decision maker towards the risk, $k > 1$ represents positive, and the risk is more inclined to represent opportunity, which may appear in some process projects. $k < 1$ is negative, indicating that the risk is destructive. $k = 1$ represents neutral, and k is usually set to 1. And we define x_{max}, x_{min} as :

$$x_{max} = inf(X), \tag{15}$$

$$x_{min} = sup(X), \tag{16}$$

$$X = \cup_{i=1}^{N} \{x|\mu_{\tilde{A}(x)} > 0\}, \tag{17}$$

where inf denotes the maximum lower bound on a set and sup denotes the minimum upper bound on a set.

On the coordinate axis, general fuzzy number $Fuzzy\_Risk$ and corresponding maximum and minimum sets are expressed as illustrated in Fig. 3. The upper bound of the intersection between maximum set and fuzzy number is called right utility, denoted by $u_M = sup_x(\mu_{\tilde{M}}(x) \wedge \mu_{\tilde{A}}(x))$ and the upper bound of the intersection between minimum set and fuzzy number is called left utility, denoted by $u_G = sup_x(\mu_{\tilde{G}}(x) \wedge \mu_{\tilde{A}}(x))$. According to the geometric relationship in Fig. 3, we calculate right utility and left utility as follows:

$$u_M = \frac{d - x_{min}}{(d-c) + (x_{max} - x_{min})}, \tag{18}$$

$$u_G = \frac{x_{max} - a}{(b-a) + (x_{max} - x_{min})}, \tag{19}$$

Finally, we calculate the total utility value u_T as exact risk value:

$$u_T = \frac{u_M + 1 - u_G}{2}, \tag{20}$$

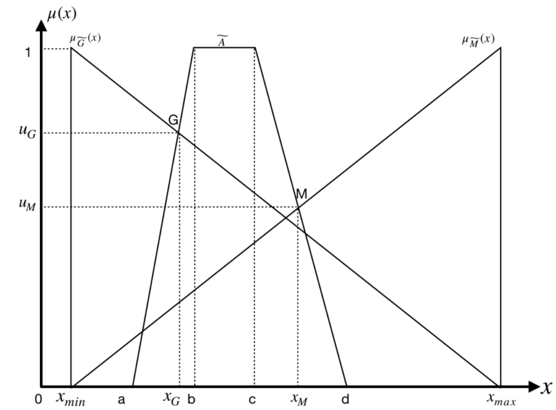

Fig. 3: The representations of fuzzy risk value and corresponding maximum and minimum sets on the coordinate axis.

Higher risk value means higher risk level of the event. According to exact risk value, the risk report shall be formed to assist trial management department to formulate the corresponding risk handling plan.

IV. Case Study Anaysis

In this section, we evaluate the performance of proposed methodology through real case study in trial scenes. We elaborate experimental settings and state-of-the-art comparison schemes in Section IV-A, then we illustrate experimental results in Section IV-B.

A. Experimental Settings and Comparison Schemes

In order to evaluate the performance of proposed method, we apply the method to a real case study in trial scene for trial risk analysis, and we construct a risk assessment model based on the method presented in Section III.

Without loss of generality, we take quality risk event analysis and public opinion risk event analysis, which are play vital roles in trial process management, as examples to evaluate the performance of proposed risk assessment method.

We also compare proposed method with state-of-the-art risk assessment methods, including methods proposed by Chutia et al. [12], Hejazi et al [13], and Ahmad et al. [14] which are mainly based on similarity measure on general fuzzy numbers.

B. Experimental Results

As mentioned in Section IV-A, we take a real trial case as an example to evaluate proposed method and state-of-the-art methods. For each risk event in the real trial case, we take three judges or law experts' risk evaluating description in three aspects as input. In this real trial case, the evaluation information matrix for public opinion risk and quality risk is present in Table II. Based on the characteristics of actual trial process, the value range of probability is extended from generalized fuzzy number to real number range, that is, "1" means that the risk event has occurred and "0" means that the risk event has not occurred at all (as shown in Table II). When a certain index is supported by relevant facts in the current

TABLE II: The evaluation information matrix for public opinion risk and quality risk in a specific trial case.

Evaluation Experts	Evaluation for Public Opinion Risk			Evaluation for Quality Risk		
	Probability	Impact	Repairability	Probability	Impact	Repairability
Expert No.1	0	H	H	1	FH	L
Expert No.2	AL	VH	FH	VH	M	VL
Expert No.3	0	VH	M	H	H	FL

TABLE III: Risk assessment results of different methods

Methods	Public Opinion Risk	Quality Risk
Proposed	**AL**	**VH**
Ahmad	H	AH
Chutia	H	VH
Hejazi	AH	VH

trial process data, there is no need to evaluate occurrence probability for corresponding risk event, which can be directly read from trial business system and automatically filled. On the other hand, if the relevant data cannot be obtain directly, the probability is then described in form of generalized fuzzy number.

According to Table I, we transform evaluation matrix in Table II. Then following the process as shown in Equation 3 to Equation 12, we aggregate multiple judges or law experts' evaluation to obtain fuzzy risk value. Finally, we calculate exact risk value as Equation 13 to Equation 20. And the risk assessment results is present in Table III. As we can see, the risk assessment results of 4 methods in terms of quality risk are basically the same. However, in the assessment of public opinion risk, the assessment results of Ahmad et al., Chutia et al., and Hejazi et al. are high-risk warning, while assessment result of the method in this paper is Absolutely Low (AL). According to Table II, the probability coefficients for public opinion risks are basically 0 which comes from relevant facts in current trial process data for this trial case, indicating that there is no public opinion risk. That is to say, the assessment results of the method in this paper are correct. The main reason is that the three existing methods assume that all the indicators must have the possibility of occurrence and ignore the conditions in the actual trial risk assessment that can directly clarify some factual situation through exiting trial process data. As a result, no matter what the actual trial situation is, all the trial case flows will be judged as being at risk. It demonstrates that our method is able to achieve accurate risk assessment and better business applicability in the complex trial scenes.

V. Conclusion

In this paper, we propose a risk assessment method for trial process management based on total utility of generalized fuzzy numbers. We first sorts out all kinds of trial risks and establish a general trial risk event database as basis for risk assessment. Then according to qualitative evaluation of law experts, we transform these evaluations into general fuzzy numbers and aggregate multiple experts' evaluations to calculate fuzzy risk values. Finally, we de-fuzzy the fuzzy risk value by calculating total utility based on Maximum and Minimum Set, obtaining an exact risk value which can provide a reliable guarantee for warning mechanism of trial risk. And the results in case study analysis demonstrate the accuracy, effectiveness and business applicability of proposed method in trial risk assessment.

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